Eigenvector (de)localization in a generalized Rosenzweig-Porter model[1]

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[1] Based on joint work with G. Cipolloni and L. Erdős.

A generalized Rosenzweig-Porter model (1960s)

Consider Hermitian random matrices $H_\lambda = H_\lambda^* \in \mathbb{C}^{N \times N}$ of the form

$$H_\lambda = H_0 + \lambda W$$

- H_0 is an *arbitrary* deterministic Hermitian matrix (also *off-diagonal*)
- W is a random Wigner matrix ($\supsetneq$ *GUE/GOE*)
- coupling parameter $\lambda = \lambda(N) > 0$ is *arbitrary*

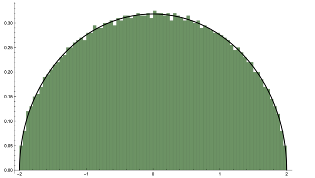
Goal: Study eigenvectors of H_λ for large N, *everywhere* in the spectrum

What are Wigner matrices?

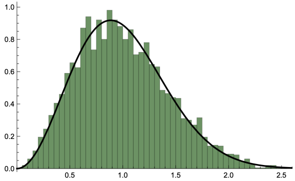
Wigner's vision: Eigenvalues of random matrices model energy levels of large disordered quantum systems

$$W = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1N} \\ w_{21} & w_{22} & \cdots & w_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & w_{NN} \end{pmatrix}$$

- w_{ij} are centered i.i.d. random variables up to $w_{ij} = \bar{w}_{ji}$
- moment bound $\mathbb{E}|\sqrt{N} w_{ij}|^p \le C_p$ and normalization $\mathbb{E}|w_{ij}|^2 = \frac{1}{N}$
- eigenvalues $\lambda_1 \le \ldots \le \lambda_N$ with ℓ^2-norm. eigenvectors $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_N$



Semicircle law $\rho_{\rm sc}(x) = (2\pi)^{-1}\sqrt{[4-x^2]_+}$



Wigner surmise $p^{(2)}_{\rm Wig.}(x) = \frac{32}{\pi^2} x^2 \mathbb{E}^{-4x^2/\pi}$

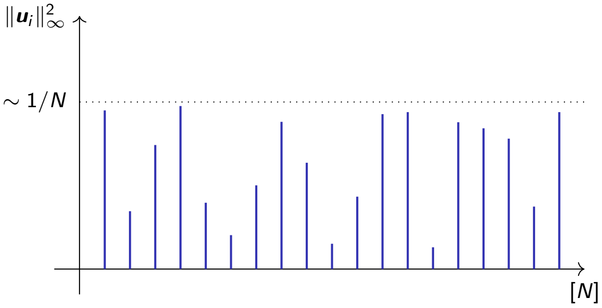
Eigenvectors of Wigner matrices I: Complete delocalization

Recall: $W\boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$ and $\|\boldsymbol{u}_i\| = 1$

Complete delocalization [Erdős-Schlein-Yau '09, ...]

For any deterministic $\boldsymbol{x} \in \mathbb{C}^N$, up to N^ϵ *with very high probability* ($\prec$):

$$|\langle \boldsymbol{u}_i, \boldsymbol{x}\rangle|^2 \prec \frac{1}{N}\|\boldsymbol{x}\|^2 \quad \text{uniformly in} \quad i \in [N]\,.$$



→ later: Knowles, Yin, Alt, Krüger, Schröder, Benigni, Lopatto, H., Riabov, ...

Eigenvectors of Wigner matrices II: ETH

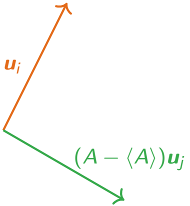
- phys: Deutsch/Srednickni '90s, therm. in closed system; mc-average
- math: Rudnick-Sarnak '90s, Quantum *Unique* Ergodicity (Q*U*E)

  $\sim$ uniform distr. of eigenfunctions (general **open problem**!)

Eigenstate Thermalization Hypothesis [C-E-H '23]

Denote $\langle R \rangle := N^{-1} \operatorname{tr} R$. For any det. $A \in \mathbb{C}^{N \times N}$ with $\mathring{A} := A - \langle A \rangle$:

$$|\langle \boldsymbol{u}_i, A\boldsymbol{u}_j \rangle - \delta_{ij} \langle A \rangle| \prec \sqrt{\frac{\langle |\mathring{A}|^2 \rangle}{N}} \quad \text{uniformly in} \quad i,j \in [N] \,.$$



Have $\boldsymbol{u}_i \perp \mathring{A}\boldsymbol{u}_j$

- previously $\|\mathring{A}\|$, and $\langle |\mathring{A}|^2 \rangle^{1/2}$ in bulk [C-E-Schröder '20-22]; HS norm much better
- $1/\sqrt{N}$-decay *uniformly* in the spectrum; without any structure $\to$ see **later**!
- related works: Benigni-Lopatto et al.

Part 1: Results

Profile function in RP model ($\sim$ spatial profile in RBM)

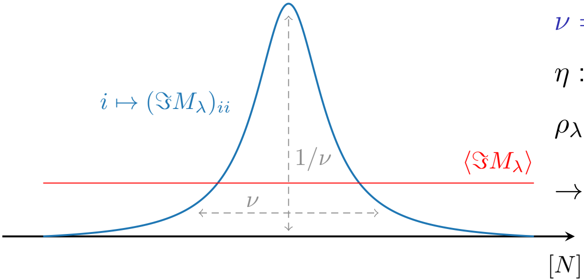
Goal: Establish analogs of delocalization and ETH in the RP model

$$H_\lambda = H_0 + \lambda W$$

Solution M_λ to **Matrix Dyson Equation (MDE)**

$$-\frac{1}{M_\lambda(z)} = z - H_0 + \lambda^2 \langle M_\lambda(z) \rangle, \qquad \Im M_\lambda \cdot \Im z > 0.$$

$\rightarrow$ from $(H_\lambda - z)^{-1} \approx M_\lambda(z)$ **(local law)**: used to approx. individual $\boldsymbol{u}_i$'s



$\nu = \eta + \lambda^2 \rho_\lambda(z)$ with

$\eta := |\Im z|$ and

$\rho_\lambda(z) := \pi^{-1} |\langle \Im M_\lambda(z) \rangle|$

$\rightarrow$ highly inhomogeneous

$$\boxed{\Im M_\lambda \not\sim \langle \Im M_\lambda \rangle}$$

norms behave very differently: $\|M_\lambda\|^2 \lesssim \frac{1}{\nu^2}$ but $\langle |M_\lambda|^2 \rangle \lesssim \frac{\rho}{\nu}$

(De)localization in the RP model

Recall: $H_\lambda = H_0 + \lambda W$ with $H_\lambda \boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$, $\|\boldsymbol{u}_i\| = 1$; $M_\lambda(z)$ solves **MDE**

Define the *quantum state*: $\Gamma_\lambda(z) := \dfrac{\Im M_\lambda(z)}{\langle \Im M_\lambda(z) \rangle}$

Theorem [C-E-H '25+]

Let $N\eta_{\mathfrak{f}}(E)\rho_\lambda(E + \mathrm{i}\eta_{\mathfrak{f}}(E)) = 1$. Then, for any deterministic $\boldsymbol{x} \in \mathbb{C}^N$

$$\big|\langle \boldsymbol{u}_i, \boldsymbol{x} \rangle\big|^2 \prec \frac{\big[\Gamma_\lambda(\lambda_i + \mathrm{i}\eta_{\mathfrak{f}}(\lambda_i))\big]_{\boldsymbol{xx}}}{N}$$

$\rightarrow$ we recover full delocalization bounds for flat $\Gamma_\lambda \sim \mathbf{1}$

- have **(i)** localized, **(ii)** non-ergodic deloc., **(iii)** fully deloc. phases
- previously: $H_0 =$ diag. and "regular", $W =$ GUE/GOE, only bulk
  $\longrightarrow$ Bourgade, Huang, Landon, Yau, von Soosten, Warzel, Benigni, ...

$$\left|\langle \boldsymbol{u}_i, \boldsymbol{x}\rangle\right|^2 \prec \frac{\left[\Gamma_\lambda(\lambda_i + \mathrm{i}\eta_{\mathfrak{f}}(\lambda_i))\right]_{\boldsymbol{xx}}}{N}$$

For $\epsilon > 0$ and $\{\boldsymbol{x}_j\}$ an ONB (e.g. the one of H_0)

$$|\mathrm{supp}_{\{\boldsymbol{x}_j\}}(\boldsymbol{u}_i)| := \inf\left\{|\mathcal{J}| : \sum_{j\in\mathcal{J}} |\langle \boldsymbol{u}_i, \boldsymbol{x}_j\rangle|^2 \geq 1-\epsilon\right\}$$

$\rightarrow \sum_j \left[\Gamma_\lambda(\lambda_i + \mathrm{i}\eta_{\mathfrak{f}}(\lambda_i))\right]_{\boldsymbol{x}_j\boldsymbol{x}_j} = N$ allows conclusion about localization

Definition

Regime	**Support**
localized	$\lvert\mathrm{supp}\rvert \sim 1$
non-ergodic deloc.	$1 \ll \lvert\mathrm{supp}\rvert \ll N$
full delocalized	$\lvert\mathrm{supp}\rvert \sim N$

Mobility edge phenomenon: $H_\lambda := \sqrt{1-\lambda^2} H_0 + \lambda W$

Take $H_0 = \mathrm{diag}$, eigenvalues sampled from an independent Wigner matrix

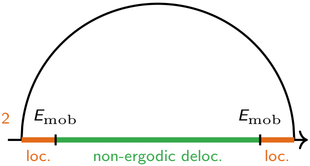
Denote $\omega_\lambda(i_0) = 1 + N\lambda^2 \rho_{i_0}^2$ with $\rho_{i_0} = \rho_{\mathrm{sc}}(\lambda_{i_0} + \mathrm{i}\eta_{\mathfrak{f}}(\lambda_{i_0}))$. Then

$$\left|\langle \boldsymbol{u}_{i_0}, \boldsymbol{e}_i \rangle\right|^2 \prec \frac{\omega_\lambda(i_0)}{\big(f_\lambda(i, i_0)\big)^2 + (\omega_\lambda(i_0))^2}$$

where for $i \approx i_0$ and $\lambda \ll 1$: $\quad f_\lambda(i, i_0) \sim (i - i_0) + N\lambda^4 \rho_{i_0} \lambda_{i_0}$

$\rightarrow$ discrete *Cauchy-like profile*

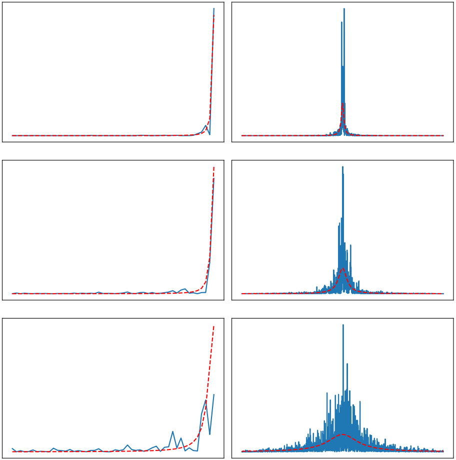
$$|\mathrm{supp}_{\{\boldsymbol{e}_i\}}(\boldsymbol{u}_{i_0})| \sim |\boldsymbol{u}_{i_0}(i_0)|^{-2} \sim 1 + N\lambda^2(4 - |\lambda_{i_0}|^2) + N\lambda^6 |\lambda_{i_0}|^2$$



For $N^{-1/2} \ll \lambda \ll N^{-1/6}$ **mobility edge** at $\big||E_{\mathrm{mob}}| - 2\big| \sim \frac{1}{N\lambda^2}$

Edge $\rho_{i_0} \sim N^{-1/3}$	**Bulk** $\rho_{i_0} \sim 1$	
$\lambda \ll N^{-1/6}$	$\lambda \ll N^{-1/2}$	localized
$N^{-1/6} \ll \lambda \ll 1$	$N^{-1/2} \ll \lambda \ll 1$	non-ergodic deloc.
$\lambda \gtrsim 1$	$\lambda \gtrsim 1$	full delocalized

Simulation: $N = 2000$, edge only vertices $1950 - 2000$



$\lambda = N^{-1/4}$

$\lambda = N^{-1/6}$

$\lambda = N^{-1/12}$

Re-entrant localization [Ghosh et al, PRB 2025]

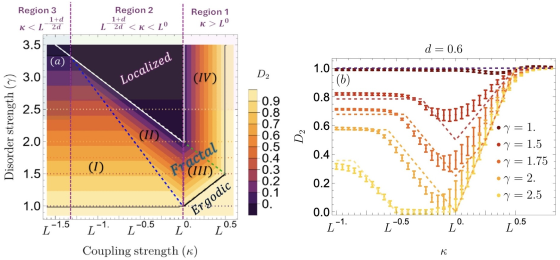
Consider specific tridiagonal Hamiltonian

$$(H_0)_{ij} := h_i \delta_{i,j} + \kappa(\delta_{i,j+1} + \delta_{i,j-1})$$

- gaps $s_\ell := h_{i_{\ell+1}} - h_{i_\ell}$ Pareto, $P(s) = \frac{d}{N\, s^{d+1}} \mathbf{1}(s \geq N^{-1/d})$, $d \in (0,1)$
- $\kappa(\delta_{i,j+1} + \delta_{i,j-1})$ discrete Laplacian

Inverse participation ratio $\mathrm{IPR}_2 = \sum_{a=1}^{N} |\boldsymbol{u}_i(a)|^4 = N^{-D_2}$ in bulk

$$\to \text{have} \quad \boxed{\mathrm{IPR}_2 = (1 + o(1)) \sum_{a=1}^{N} \left([\Gamma_\lambda(\lambda_i)]_{aa} \right)^2} \quad (\mathrm{supp\ in\ } \boldsymbol{e}_i\text{-basis!})$$



Translation:

- $N \to L$
- $\lambda \to N^{(1-\gamma)/2}$

Our H_0 is arbitrary, we provide rigorous proof.

Eigenstate Thermalization Hypothesis: Variance function

Recall M_λ solution of MDE and $\Gamma_\lambda = \Im M_\lambda / \langle \Im M_\lambda \rangle$.

$$M_i := M_\lambda(\widehat{\lambda}_i) \qquad \Gamma_i := \frac{\Im M_i}{\langle \Im M_i \rangle} \quad \text{with} \quad \widehat{\lambda}_i := \lambda_i + \mathrm{i}\eta_{\mathfrak{f}}(\lambda_i)$$

Regularization of $A \in \mathbb{C}^{N \times N}$, removes singular part of A ($\sim$ traceless):

$$\mathring{A}^{i,j} := A - \mathbf{1}(i \approx j) \frac{\langle M_i A M_j^* \rangle}{\langle M_i M_j^* \rangle} \mathbf{1}$$

Variance functional for $i \approx j$, other regime similar (simpler); $\mathring{A} \equiv \mathring{A}^{i,j}$

$$\big(\mathfrak{s}_2(i,j;A)\big)^2 := \langle \Gamma_i \mathring{A} \Gamma_j \mathring{A}^* \rangle + \lambda^2 \left| \frac{\langle M_i \mathring{A} \Gamma_j \rangle \langle \Gamma_i \mathring{A}^* M_j \rangle}{1 - \lambda^2 \langle M_i M_j \rangle} \right|$$

$\rightarrow$ **inhom. HS norm** of A measures overlap with the M-profiles at λ_i, λ_j

Eigenstate Thermalization Hypothesis [Deutsch, Srednicki]

Consider $H_\lambda \boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$ with $\|\boldsymbol{u}_i\| = 1$

Theorem [C-E-H '25+]

For any deterministic $A \in \mathbb{C}^{N\times N}$, *uniformly* in $i, j \in [N]$:

$$\boxed{\left|\langle \boldsymbol{u}_i, A\boldsymbol{u}_j\rangle - \delta_{ij}\langle \Gamma_i A\rangle\right| \prec \frac{\mathfrak{s}_2(i,j;A)}{\sqrt{N}}}$$

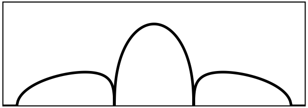
$\to \langle \Gamma_i A\rangle = \frac{\langle \Im M_i A\rangle}{\langle \Im M_i\rangle} \approx$ microcanonical average at energy λ_i

(i) Take $A = |\boldsymbol{x}\rangle\langle \boldsymbol{x}|$ $\Longrightarrow$ delocalization result

(ii) recover Wigner ETH as special case since:

$$\Gamma_i = \mathrm{Id} \quad \text{and} \quad \mathfrak{s}_2 = \langle |A - \langle A\rangle|^2\rangle^{1/2}$$

Anomalous decay in ETH



(iii) for $\lambda = 1$ with flat $\Im M \sim \langle \Im M \rangle$

$$\left|\left\langle \boldsymbol{u}_i, A\boldsymbol{u}_j \right\rangle - \delta_{ij}\langle \Gamma_i A\rangle\right| \prec \frac{\langle |\mathring{A}^{i,j}|^2\rangle^{1/2}}{\sqrt{N}} \times \frac{1}{|1-\langle M_i M_j\rangle|^{1/2}}$$

uniformly in the whole spectrum involving stability factor

- previously only in the bulk with $\|\mathring{A}^{i,j}\|$ [C-E-H-Kolupaiev '23]
- *new anomalous ETH decay*: $(N|1-\langle M_i^2\rangle|)^{-1/2} \sim \begin{cases} N^{-1/2} & \text{bulk} \\ N^{-1/3} & \text{edge} \\ N^{-1/4} & \text{cusp} \end{cases}$

  studied in full generality for correlated matrices [E-H-Riabov '25+]

$\rightarrow$ remove extra term $\left|\left\langle \boldsymbol{u}_i, A\boldsymbol{u}_j\right\rangle - \delta_{ij}\langle\Gamma_i A\rangle - \langle L_{ij}A\rangle\langle \boldsymbol{u}_i, R_{ij}\boldsymbol{u}_j\rangle\right|$ for $N^{-1/2}$

Part 2: Elements of the proof

Delocalization and ETH from resolvent bounds

Workhorse: control on resolvent $G(z) := (H_\lambda - z)^{-1}$ of H_λ

Single resolvent bound

For $z \in \mathbb{C} \setminus \mathbb{R}$, let $\eta := |\Im z|$ and $\rho(z) := \pi^{-1}|\langle \Im M(z) \rangle|$. For $N\eta\rho(z) \gtrsim 1$:

$$|(\Im G(z))_{\boldsymbol{xx}}| \prec \rho(z)[\Gamma(z)]_{\boldsymbol{xx}} \qquad \text{uniformly in} \quad \boldsymbol{x} \in \mathbb{C}^N$$

Pick $z_{i_0} = \lambda_{i_0} + \mathrm{i}\eta_{\mathfrak{f},i_0}$. Then:

$$\frac{|\langle \boldsymbol{u}_{i_0}, \boldsymbol{x} \rangle|^2}{\eta_{\mathfrak{f},i_0}} \leq \sum_i \frac{|\langle \boldsymbol{u}_i, \boldsymbol{x} \rangle|^2 \eta_{\mathfrak{f},i_0}}{(\lambda_i - \lambda_{i_0})^2 + (\eta_{\mathfrak{f},i_0})^2}$$

$$= (\Im G(z_{i_0}))_{\boldsymbol{xx}} \prec \rho(z_{i_0})[\Gamma(z_{i_0})]_{\boldsymbol{xx}}$$

$\to$ Since $N\eta_{\mathfrak{f},i_0}\rho(z_{i_0}) = 1$, we get $|\langle \boldsymbol{u}_{i_0}, \boldsymbol{x} \rangle|^2 \prec [\Gamma(z_{i_0})]_{\boldsymbol{xx}}/N$

Rmk: Prove not just bounds, but *local laws* (= concentration estimates)

Two resolvent bound

For $z_i \in \mathbb{C} \setminus \mathbb{R}$, $i = 1, 2$, let $\eta_i := |\Im z_i|$ and $\rho_i := \pi^{-1} |\langle \Im M(z_i) \rangle|$. Then,

$$|\langle \Im G(z_1) \mathring{A} \Im G(z_2) \mathring{A}^* \rangle| \prec \rho_1 \rho_2 \big(\mathfrak{s}_2(z_1, z_2; A) \big)^2$$

for $N \min_i(\eta_i \rho_i) \gtrsim 1$, where $\mathring{A} = \mathring{A}^{z_1, z_2}$ is the *regularized observable*

Rmk: Regularization important, improves naive size by η; ρ_i's hard to gain

Pick $z_{i_0} = \lambda_{i_0} + \mathrm{i}\eta_{\mathfrak{f}, i_0}$ and $z_{j_0} = \lambda_{j_0} + \mathrm{i}\eta_{\mathfrak{f}, j_0}$. Then:

$$\frac{|\langle \boldsymbol{u}_{i_0}, \mathring{A} \boldsymbol{u}_{j_0} \rangle|^2}{N \eta_{\mathfrak{f}, i_0} \eta_{\mathfrak{f}, j_0}} \le \frac{1}{N} \sum_{i,j} \frac{|\langle \boldsymbol{u}_i, \mathring{A} \boldsymbol{u}_j \rangle|^2 \eta_{\mathfrak{f}, i_0} \eta_{\mathfrak{f}, j_0}}{\big[(\lambda_i - \lambda_{i_0})^2 + \eta_{\mathfrak{f}, i_0}^2\big] \big[(\lambda_j - \lambda_{j_0})^2 + \eta_{\mathfrak{f}, j_0}^2\big]}$$

$$= \langle \Im G(z_{i_0}) \mathring{A} \Im G(z_{j_0}) \mathring{A}^* \rangle \prec \rho(z_{i_0}) \rho(z_{j_0}) \big(\mathfrak{s}_2(i_0, j_0; A) \big)^2$$

$\rightarrow$ Since $N \eta_{\mathfrak{f}, i_0} \rho(z_{i_0}) = N \eta_{\mathfrak{f}, j_0} \rho(z_{j_0}) = 1$, we get ETH.

Zigzag strategy (present only $1G$ Wigner but *very* robust)

Three steps: (i) global law, (ii) **Zig**: characteristic flow, (iii) **Zag**: GFT

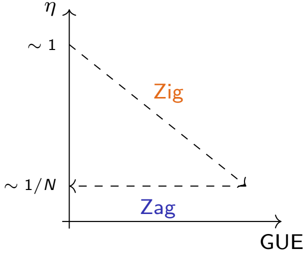
(i): Initial condition: For $\eta \sim 1$, have $|\langle G - m\rangle| \prec 1/N$. Affordable: $\|G_i\| \lesssim 1$, simplified independent proof

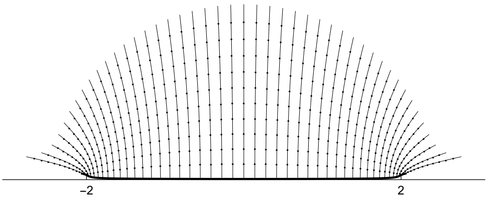
(ii): For $G_t(z_t) = (W_t - z_t)^{-1}$ show $|\langle G_t(z_t) - m(z_t)\rangle| \prec \frac{1}{N\eta_t}$ with

$$\mathrm{d}W_t = -\frac{1}{2}W_t\mathrm{d}t + \frac{\mathrm{d}B_t}{\sqrt{N}}, \quad W_0 = W, \quad \text{and} \quad \boxed{\partial_t z_t = -m(z_t) - \frac{z_t}{2}}$$

$\rightarrow$ characteristics cancel critical term in Ito formula for $\mathrm{d}\langle G_t(z_t)\rangle$

(iii): Remove Gaussian comp. by self-cons. GFT (Gronwall); disc./cont.





Main challenges

- Carry spatial 1-point and 2-point control parameters $[\Gamma(z)]_{xy}$ and $\mathfrak{s}_2(z_1, z_2; A)$ through the proof; previously in mean-field both *flat* $\to$ similar (complicated) profiles for RBM [Yau-Yin, E-Riabov]

- H_0 arbitrary: lack of eff. perturbation theory; proof strictly local in z's

$\to$ linearization $\quad G(z_1)G(z_2) = \frac{1}{2\pi \mathrm{i}} \oint_\gamma \frac{G(w)}{(w-z_1)(w-z_2)} \mathrm{d}w \quad$ not applicable

Comments on Zigzag steps

(i) *Global law:* bootstrap from N^{1000} down to $\lambda^2\rho/\eta \gtrsim 1$

(ii) *Zig:* prop. to $\lambda^2\rho/\eta \ll 1$, $\langle \Im GA \Im GA^* \rangle$ & $\langle GAG \rangle$ are *evolved together*

(iii) *Zag:* good Schwarz inequ. avoid 3-point fncts., e.g. 3^{rd} ord. cumulant:

$$\frac{\lambda^3}{N^{5/2}} \sum_{a,b} G_{aa}(\Im G)_{bb}(GA\Im | GA^* G^*)_{ab} \lesssim \frac{\lambda^3}{N^{5/2}} \sqrt{\sum_{a,b} |M_{ab}|^2} \sqrt{\sum_b |(\Im M)_{bb}|^2} \sum_a (GA\Im GA^* G^*)_{aa}$$

(use Ward + M bounds) $\quad \lesssim \sqrt{\frac{\lambda^2\rho}{\eta}} \frac{\langle \Im GA \Im GA^* \rangle}{\sqrt{N\eta\rho}} \ll \frac{1}{t_{\mathrm{zag}}} \frac{\langle \Im GA \Im GA^* \rangle}{\sqrt{N\eta\rho}} \quad \to$ Gronwall

Further results provable using fine local laws

Our method(s), in principle, allow studying further questions on H_λ, e.g.:

- local spectral universality in "mini-bulks" [Péché]
- Gaussian fluctuations of $|\langle \boldsymbol{u}_i, \boldsymbol{x}\rangle|^2$ and $\langle \boldsymbol{u}_i, A\boldsymbol{u}_i\rangle$ (in the bulk), using DBM analysis of eigenvectors [Bourgade, Yau, Benigni, Marcinek, ...]
- quantum diffusion: $H_0 = \mathrm{diag}(\mu_1, ..., \mu_N)$ density $\rho_0 =: \frac{\alpha}{2\pi}$ at μ_{i_0}

$$\frac{1}{|\mathcal{J}|}\sum_{j\in\mathcal{J}}\left|\left(\mathrm{e}^{\mathrm{i}tH_\lambda}\right)_{i_0 j}\right|^2 \approx \left(1-\mathrm{e}^{-\alpha\lambda^2 t}\right)\frac{\sum_{j\in\mathcal{J}}\left((\mu_j-\mu_{i_0})^2+(\alpha\lambda^2)^2\right)^{-1}}{|\mathcal{J}|\sum_{j\in[N]}\left((\mu_j-\mu_{i_0})^2+(\alpha\lambda^2)^2\right)^{-1}} + \mathrm{e}^{-\alpha\lambda^2 t}\frac{\mathbf{1}(i_0\in\mathcal{J})}{|\mathcal{J}|}$$

  for $|\mathcal{J}| \sim N$ using $\mathrm{e}^{itH_\lambda} = \frac{1}{2\pi\mathrm{i}}\oint \mathrm{e}^{\mathrm{i}tz}G(z)\mathrm{d}z$; cf. [E-H-Reker-Riabov '23]

- analysis of spectral form factor $\mathrm{SFF}(t) = \mathbb{E}\left|\langle \mathrm{e}^{\mathrm{i}tH_\lambda}\rangle\right|^2$ for short times

Summary: (De)localization in RP model

$$H_\lambda = H_0 + \lambda W$$

(i) Generalized RP model, arbitrary H_0, arbitrary coupling $\lambda > 0$, general Wigner matrix W, uniformly in the spectrum

(ii) Eigenvector **(de)localization** : Mobility edge, re-entrant localization

(iii) **Eigenstate Thermalization Hypothesis** with anomalous decay

(iv) Proof via resolvent bounds, obtained by dynamical Zigzag strategy

Thanks for your attention!